Seepage Exit Gradients

1 Introduction

Most Soil Mechanics text books present and discuss the concept of seepage exit gradients and state that the exit gradients should not be greater than 1.0 Applying this criteria to two-dimensional finite element seepage analyses requires an understanding as to the conditions for which the criteria was developed and the physical meaning of an exit gradient greater than unity (1.0).

This document discusses the background as to how the exit gradient criterion was developed and how this criterion should be viewed when interpreting 2D finite element seepage analyses.

2 One-dimensional upward flow

The concept of exit gradients was developed primarily from pure upward flow in a column of soil. Consider the column in Figure 1. Water will flow upward through the column if the applied total hydraulic head *H* at the bottom of the column is greater than the surface elevation of the column.

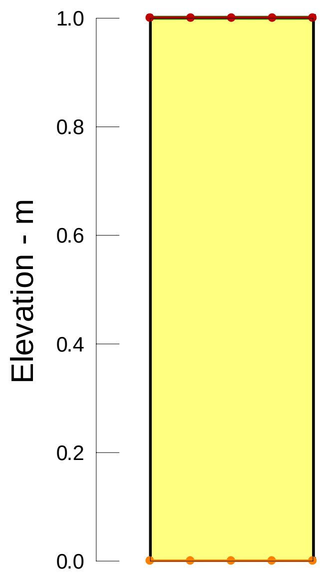

Figure 1 One-dimensional soil column

If for example *H* at the base is specified as 1.2 m water will flow upward through the column. The total head loss is 1.2 minus 1.0 which equals 0.2 m of head. The gradient is the total head loss divided by the height (length) of the column which in this case is 0.2. In equation form,

$$i = \frac{\left(H_{base} - H_{top}\right)}{L} = \frac{(1.2 - 1.0)}{1.0} = 0.2$$

For discussion purposes, let us assume that the total unit weight of the soil is 20 kN/m^3 and that the unit weight of the water is 10 kN/m^3.

Now if we apply a total head at the base of the column of equal to 2 m (equals 2 m of pressure head since the elevation at the base is zero), the upward gradient will be 1.0 and the effective stress throughout the column will be zero.

$$\sigma' = (\sigma - u) = (\gamma h - \gamma_w H) = (20*1 - 10*2) = 0$$

When the effective stress is zero, the gradient is sometimes referred to as the critical gradient, and since the zero effective stress conditions occur when the gradient is 1.0, the critical gradient is 1.0.

The zero effective stress condition under upward flow conditions is also referred to as a "quick" condition. It is this condition that is referred to colloquially sometimes as "quick sand" or "boiling."

Figure 2 shows the SEEP/W computed gradients when *H* at the bottom of the column is 2.0 and 1.0 m at the top of the column in Figure 1. This matches the earlier hand calculations.

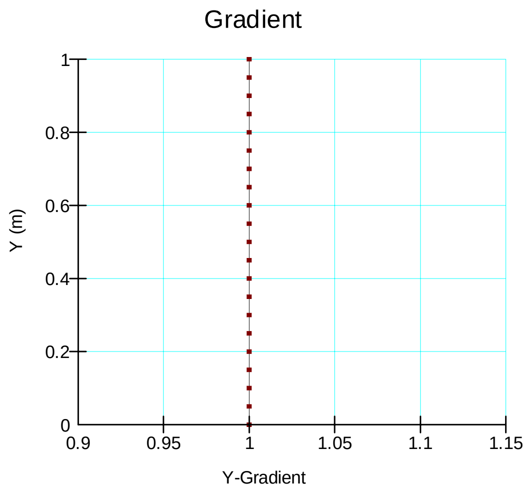

Figure 2 SEEP/W computed gradients

An important observation in the context of the later 2D flow discussions is that the gradient is a constant throughout the column.

The criterion that the exit gradient should not exceed 1.0 comes from this type of 1D upward flow analysis.

3 Two-dimensional flow under at hydraulic structure

Figure 3 shows a typical flow net seepage solution for flow under a hydraulic structure. The example is taken from the text book *Soil Mechanics, SI Version* by T.W. Lambe and R.V. Whitman, published by John Wiley & Sons.

This example illustrates how the exit gradient is computed from a flow net. The upward gradient is computed in the area marked with an X. The total head loss *H* between the last two equipotential lines is 0.62 m. The distance between the two equipotential lines on the downstream end in the X area is 3.3 m. The exit gradient is then computed as 0.62 divided by 3.3 making the upward gradient 0.19.

Of significance for later discussions is that the gradient is computed for a head loss over a fairly long distance of 3.3 m. The computed gradient is in essence an average gradient over this distance. Recognizing this as an average over a significant distance is important when we later discuss finite element results.

▶ **Example 18.2**

Given: Flow net in Fig. E18.2

Find: Pressure heads at points *A* to *H*; quantity of seepage; gradient in *X*

Solution: The pressure heads are shown in Fig. E18.2.

Seepage:

$$n_f = 4 \qquad n_d = 12.6 \qquad k = 0.05 \text{ cm/sec} \qquad \S = \frac{n_f}{n_d} = 0.317$$

$$\frac{Q}{L} = kH\S = 0.05 \times 10^{-2} \text{ m/sec} \times 7.8 \text{ m} \times 0.317 = 0.00124 \text{ (m}^3\text{/sec)(m)}$$

Gradient in *X*:

$$i_X = \frac{\Delta h}{l} = \frac{0.62}{3.3} = 0.19$$

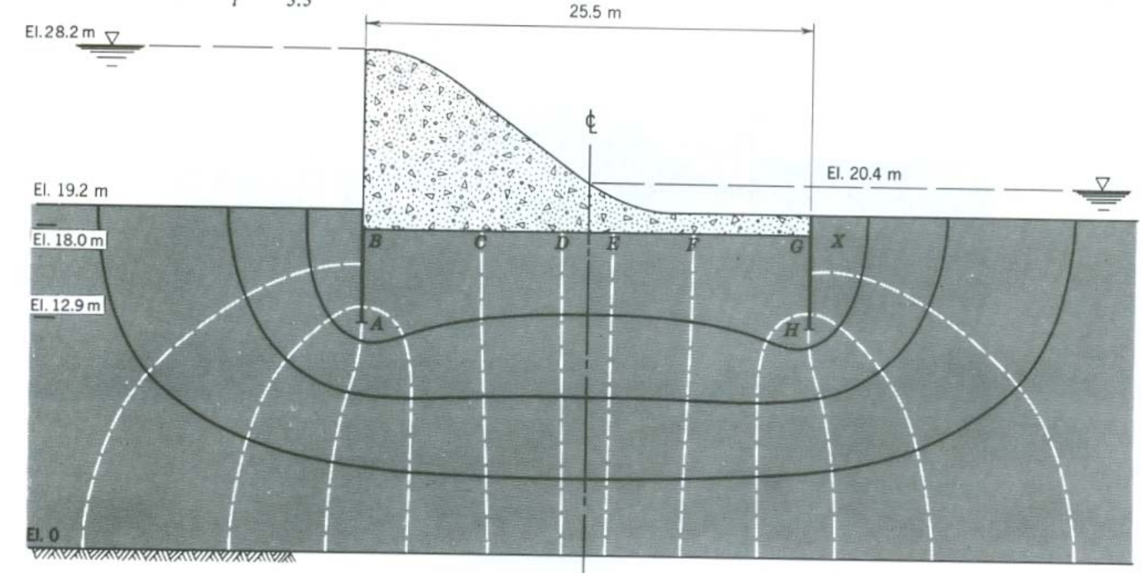

Figure 3 Example flow net from the Lambe and Whitman text book, page 271

Figure 4 shows the SEEP/W solution for the above text book example.

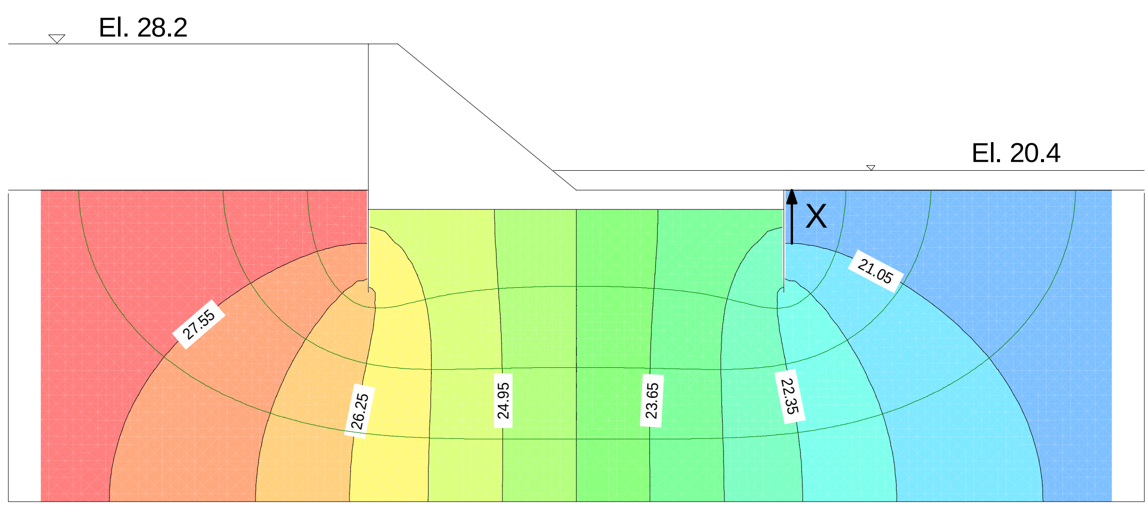

Figure 4 SEEP/W solution for the text book example

Now when we compute the upward gradient in the X area, the total head loss is between the last two equipotential lines is 21.05 - 20.4 = 0.65 (the equipotential increments are not all even in the text book example – there is about a half increment around the center-line. SEEP/W cannot accommodate this; the increments must all be the same and consequently the increment is 0.65 instead of 0.62 in Figure 3). The vertical distance at the location of the upward arrow in Figure 4 is about El 19.2 – 15.9 = 3.3 m. The upward gradient is therefore 0.65/3.3 = 0.20 which is in essence the same as for the flow net case.

In a finite element formulation the gradients are computed at what are known as Gauss integration points. In the context of local element coordinates which are ±1.0 at the corners of the element and (0, 0) at the center of a quadrilateral element, the Gauss integration points for a 4-noded quadrilateral are at local coordinates equal to ± 0.577. In SEEP/W we shade Gauss integration areas as shown in Figure 5. The actual Gauss integration point is about in the middle of the Gauss area. The Gauss area is shown primarily for convenient reference. It is important however to recognize that the calculations are done at the Gauss integration point. This is discussed in more detail in the theory chapters of the related GeoStudio Engineering Books.

Results can be inspected at the Gauss integration points with the View Results Information command in CONTOUR by clicking on a Gauss region.

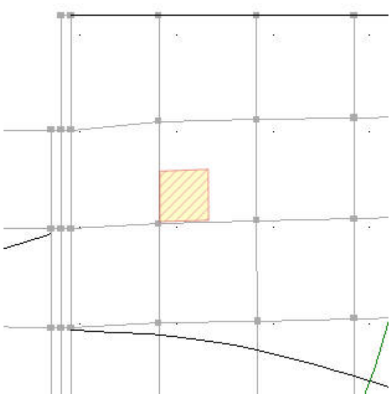

Figure 5 Illustration of a SEEP/W Gauss region

If we now click on the Gauss regions beside the up-arrow in the X area in Figure 4, the y-gradients vary between 0.188 and 0.206. The variation is small because the flow is predominantly upward and represents the situation discussed earlier for 1D upward flow in a column. However while the variation is small, it nonetheless demonstrates that the flow net method of computing exit gradients is an average over a meaningful distance.

Gradients can also be inspected at nodes. The gradients at nodes are however not computed as part of the finite element solution. They are computed as the average of the gradients in all the Gauss regions common to a node or that touch a node. Computing gradients at the nodes is done primarily for contouring purposes.

4 Homogeneous isotropic case

The exit gradient is independent of the hydraulic conductivity if the soil is homogeneous and isotropic ($K_x = K_y$). This is evident by examining the partial differential equation for steady-state seepage flow.

$$k_x \frac{\partial^2 h}{\partial x^2} + k_y \frac{\partial^2 h}{\partial y^2} = 0$$

If K_x is equal to Ky we can divide both sides of the equation by K_x and get,

$$\frac{\partial^2 h}{\partial x^2}+\frac{\partial^2 h}{\partial y^2}=0$$

This shows that the pressure distribution is independent of the hydraulic conductivity in this special case and therefore the exit gradient is independent of the hydraulic conductivity.

The specific discharge or Darcian velocity (called Liquid Velocity in SEEP/W) is however directly related to the hydraulic conductivity. For fairly permeable sands the rate of upward flow will be much higher than for less permeable silty materials even though the exit gradient is the same. The point of significance here is that when we consider the possibility of piping where seepage exits the flow system it is necessary to look at more than just exit gradients – flow quantities and velocities also need to be considered.

5 Two-dimensional flow

Now let us look at the case of a simple 1:1 slope as illustrated in Figure 6. The ground surface profile has a sharp corner at the slope crest and at the slope toe. The purpose here is to examine the exit gradient at the slope toe.

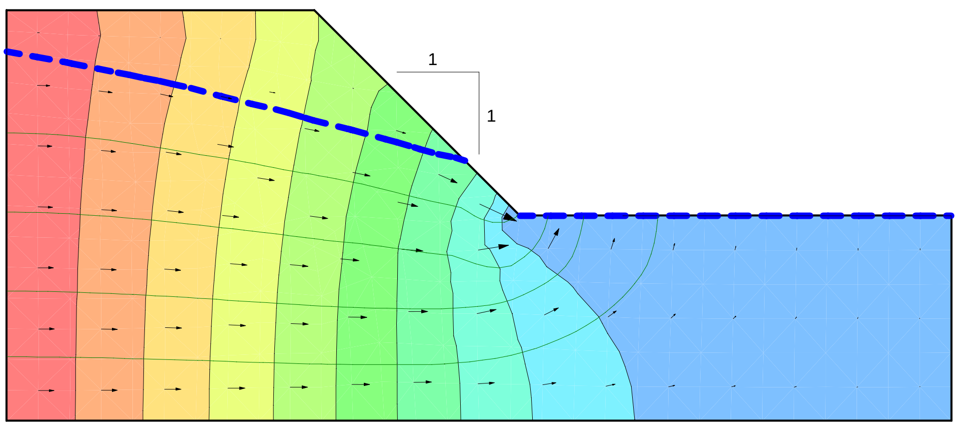

Figure 6

In mathematical terminology a sharp corner in the ground surface like at the slope toe is known as a point of singularity. Fundamentally, this means that the solution to the partial differential equation describing the flow within the system is undefined at the point of singularity. In this case it means that the derivatives of the flow equations are discontinuous at the slope toe. The consequence is that the gradients (derivatives) of the flow equations tend towards infinite at the point of singularity.

These tendencies for the gradients to become ever greater as the computations move closer to the singularity point can be illustrated by looking at the results for various element sizes. The Gauss integration point moves closer to the point of singularity as the element size gets smaller.

Figure 7 shows the mesh in the toe area when the elements are about 1 m in size. The computed x-y gradient at the toe node is 0.982. This is the average of the three Gauss regions common to this node.

When the element size is reduced to about 0.4 m as in Figure 8, the x-y gradient at the toe is 1.66. Reducing the element size to about 0.1 as in Figure 9 increases the x-y gradient at the toe to 3.443.

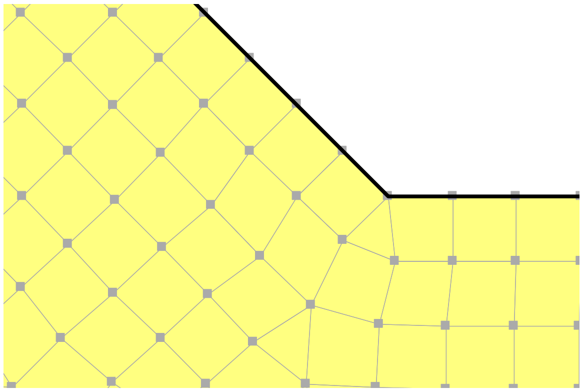

Figure 7 Mesh at toe with elements about 1 m in size

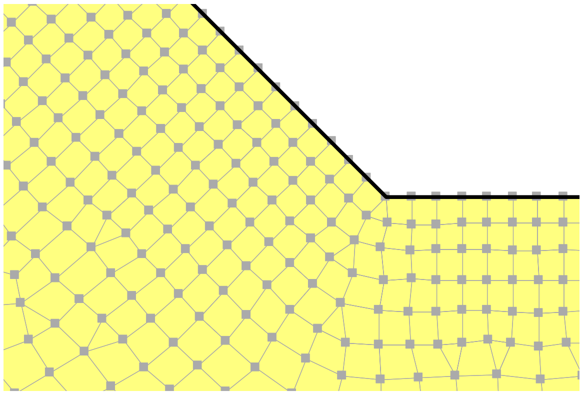

Figure 8 Mesh at the toe with elements about 0.4 m in size

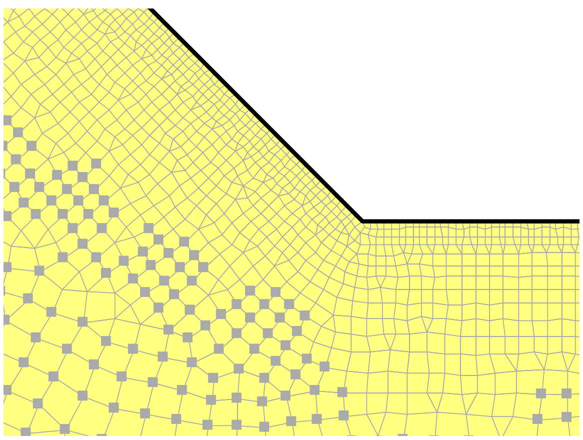

Figure 9 Mesh at the toe with elements about 0.1 m in size

Another way to inspect what happens at the point of singularity is to look at the rate of change of the gradient with distance around the toe. Figure 10 shows contours of the x-y gradients around the toe. The last (highest) contour shown is 1.0 – the value at the toe is 3.44. Note how the distance between each contour interval rapidly decreases towards the slope toe which is reflective of the rapid change in the gradient as the solution moves towards the point of singularity.

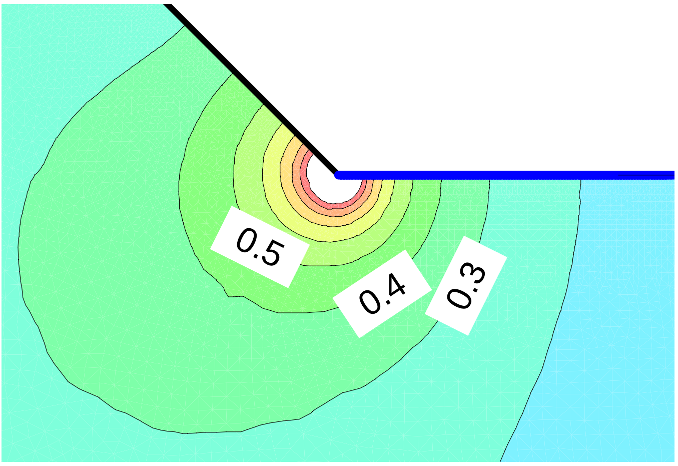

Figure 10 Contours of the x-y gradient – last contour shown is at 1.0

6 Effect of a curved transition

Theoretically as noted earlier the solution to the partial differential flow equation is undefined at points of singularity. The practical implication is that the computed gradients at points of singularity have no physical significance.

Moreover, points of singularity seldom if ever exist in real field situations (except maybe at the corners of concrete structures). If all likelihood the toe area of the slope is curved maybe something like what is illustrated in Figure 11. Now the maximum computed gradient is about 0.7 as opposed to the peak gradient of 3.44 in Figure 10. This is more realistic and further demonstrates that computed gradients at sharp points in the ground surface profile likely do not represent field conditions.

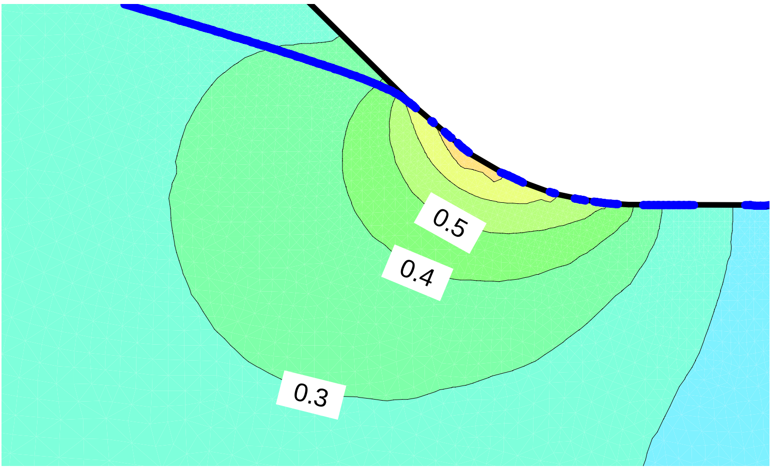

Figure 11 Gradient contours with a curved transition between the slope and the flat ground

In a large field problem it will not always be easy to make nice curves at the transition points. Using straight line segments make the model definition much easier. From a practical modeling perspective it is better to use straight line segments and ignore the exit gradients at sharp breaks in the ground surface profile then attempting to create a curve at the break points. Creating too much geometric complexity at the break points can sometimes obscure the interpretation of the overall global flow system.

7 Interpretation of SEEP/W computed exit gradients

Interpreting the exit gradients computed in a SEEP/W finite element analysis requires some judgment and understanding by the analyst. The computed results cannot always just be accepted at face value as being representative of field conditions. The follow are some issues that the analyst needs to understand and judge when interpreting the results.

1. The common belief that exit gradients should not exceed 1.0 was developed for one-dimensional upward flow which represents a "quick" condition.

2. Upward flow conditions that result in an exit gradient of 1.0 represent a zero effective stress condition. The effective stress state condition is a much more important issue than the gradient. Internal to an earth structure the gradient maybe greater than 1.0 but the effective stress may be fairly high. A gradient greater than 1.0 in such a case does not represent a "quick" condition and will therefore not necessarily be damaging to the structure.

3. Exit gradients are not necessarily the only governing issue – flow quantities also need to be considered in the context of the erodibility of the soil.

4. A seepage face is always an undesirable characteristic of engineered earth structures regardless of the exit gradients computed from a SEEP/W analysis. Generally some form of a granular protection layer is ideally required. Examining granular filter criteria is beyond the scope of this discussion.

5. When examining the exit gradients from an analysis such as the 45-degree slope illustrated earlier it is better to look at the gradients some distance away from sharp breaks in the ground surface profile, say a meter or two away. As noted earlier, the computed gradients at points of singularity are in all likelihood not reprehensive of actual field conditions anyway.

6. If you want to interpret SEEP/W exit gradients in the context of what has traditionally been done with flow nets, then you should look at the head loss between two equipotential lines over a significant distance and compute an average exit gradient of sorts.

7. Attempting to curve the ground profile with too much geometric complexity is not recommended as noted in the previous section. From a modeling prescribe it is better to simply ignore the computed gradients at sharp breaks in the ground surface profile.

8 Concluding remarks

The exit gradients computed with a SEEP/W analysis cannot always be taken at face value as being representative of the actual field conditions. Considerable understanding and judgment is vital to interpreting and using the computed results.