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The Limitations of the
Strength Reduction Approach

Why I don't like the strength-reduction approach for stability analysis

A concept has come into geotechnical engineering for assessing the stability of earth structures where the soil strength is reduced in an elastic-plastic finite-element or finite difference analysis until it is not possible to obtain a solution. The lack of a solution is deemed to indicate that the structure has reached a point of instability. A ratio of the actual soil strength to the reduced strength required to reach the point of instability is then regarded as the safety factor.

This approach has caused some considerable consternation and confusion as illustrated by a recent discussion on the [Eng-Tips Forums](#).

The strength reduction concept is being portrayed by some as the future of geotechnical stability analysis, and the inference is that it will supplant traditional limit equilibrium methods. We need to however take a careful look at the concept before jumping blindly onboard. It has some inherent limitations that affect its usefulness in practice which is why I don't like the approach.

In an elastic-plastic analysis an iterative scheme is required to re-distribute the stresses in the soil such that no element of soil is over-stressed; that is, no element of soil has a stress state that exceeds its strength. When it is no longer possible to meet these criteria in the iterative process, it is impossible to achieve a converged solution scheme. Ideally, this happens when the structure is no longer stable or the factor of safety is below unity (1.0).

So in the end a "non-solution" is the "solution". This is very unconventional in engineering design analyses; generally, we rely on converged solutions, not un-converged solutions. Intuitively, it is somewhat unsettling to rely on a non-converged solution for design.

Furthermore, it is very difficult to determine the exact point at which the instability occurs based on converged versus non-converged solutions. The reason for this is that achieving convergence becomes exponentially more difficult as the solution migrates towards the point of instability. Finer and finer incremental changes are required to maintain convergence. Also, there can be many other reasons for the lack of convergence. It may be that the strength reduction steps are too large, it could be incorrect boundary conditions or unrealistic initial insitu stresses or even inappropriate discretization. Considerable numerical analysis effort and skill is required to determine that exact point of instability from the results of finite-element analyses. This seriously affects the use of this concept in practice.

A second serious limitation of the strength reduction method and why I don't like it, is that the soil strength is reduced throughout the earth structure by an equal amount. In other words, the local factor of safety is taken to be constant along the entire slip surface. This is not the case in reality. The local factor of safety in fact varies significantly along a potential slip surface. Also, the strength of all the different soil stratigraphic units within a particular case is reduced by the same factor which is also not realistic. Furthermore, assuming that the local factor of safety is constant along a potential slip surface can lead to unrealistic stress distributions as is often the case in traditional limit equilibrium (LE) formulations. Keeping the local factor of safety constant along the entire slip surface is an inherent assumption in LE methods and in this respect the strength reduction method is no advancement beyond the LE formulations.

In my view, a better way of using finite-element computed stresses to assess margins of stability is to use the FE stresses to establish the normal stress distribution along a potential slip surface from which the available shear resistance can be computed. The FE stresses can also be used to compute the mobilized shear (driving force) along the slip surface. A ratio of the total resisting forces divided by the total mobilized shearing forces provides a safety factor. The advantage of this approach is that there is no need to struggle with converged versus non-converged solutions. Even a linear-elastic analysis which has no convergence issues at all is adequate in many cases. Moreover, the local factors of safety along the slip surface can vary in order to maintain strain compatibility in the stress distribution solution. An argument against summing the resisting and mobilized shear forces along trial slip surfaces is that there is then a need to make assumptions about the position and shape of the trial slip surfaces. In the past this was perhaps a valid argument but not any longer. The position and shape of the critical slip surface can be determined directly from the stress state in the ground using dynamic programming or optimization techniques such as those used in [SLOPE/W](#). Using these techniques, the critical surface does not need to have any predetermined geometric shape.

Granted, there are cases when an earth structure deforms to the extent that it is not able to serve its intended function long before reaching the point of total collapse. In such cases the issue is serviceability, not stability. The issue is the amount of deformation, not the factor of safety. The strength reduction method is attractive for such cases. It makes it possible to answer questions such as, "how much will the structure deform if the strength happens to fall by a certain amount."

The strength reduction approach is also attractive if indeed there has been a strength loss in the field due to, for example, elevated pore-pressures and the strength loss of strain-softening materials associated with earthquake shaking or chemical changes associated with seepage which in turn has reduced the strength of the material. The key question to ask would then be, "has there been or potentially could there be an actual loss of strength in the field with time." If the answer is yes, then the strength reduction concept is appropriate for assessing the related deformations.

In summary, finite-element or finite-difference computed stresses should not be used in isolation to compute safety factors for earth structures using the strength reduction approach. Finding the exact point of instability (F of $S = 1.0$) is too difficult and fraught with uncertainty, and it does not simulate actual processes in the field. A much better way is to use the numerically computed stresses to establish the resisting and driving shear forces

along a potential slip surface and then use the summation of these forces to compute a safety factor.

The tight product integration available in the [GeoStudio](#) suite of software makes this a relatively straightforward task.

At least, that is how I see it.

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