

## Slip Surface Exit Angles in a Limit Equilibrium Analysis

There has been some discussion surrounding the issue of slip surface exit angles in a limit equilibrium analysis. SLOPE/W limits the inclination to a maximum of  $45^\circ$ . This note discusses the issue and explains why SLOPE/W has a restriction on the exit angle.

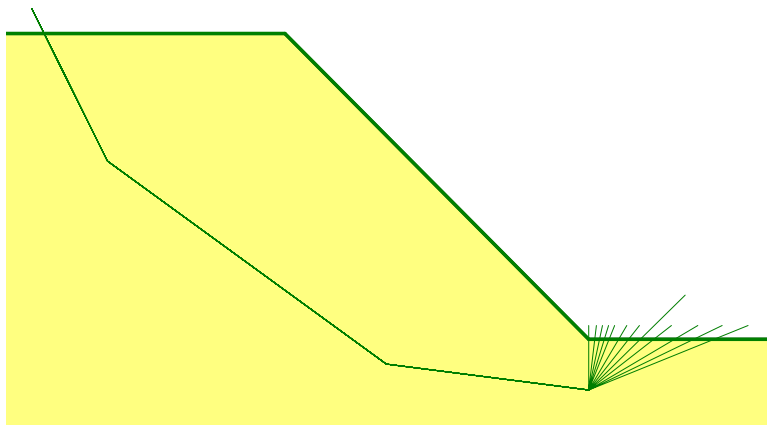
### Earth pressure considerations

From earth pressure theory, a slip surface should be inclined at  $(45^\circ + \phi/2)$  for active conditions and  $(45^\circ - \phi/2)$  for passive conditions. In a slope stability analysis the crest (or entrance area) is analogous to a zone of active pressure conditions and the toe (or exit area) is analogous to a zone of passive pressure conditions. Based on this, the entrance of the slip surface should be inclined at an angle around  $(45^\circ + \phi/2)$  and the slip surface should exit at an angle around  $(45^\circ - \phi/2)$ . If  $\phi$  (phi) is  $30^\circ$  then the entrance angle will be around  $60^\circ$  and the exit angle will be around  $30^\circ$  from a horizontal line.

In field cases when the slip surface inclination in the crest area is greater than  $(45^\circ + \phi/2)$ , there will likely be a tension crack. SLOPE/W has an option that can be selected by the user to handle this case. For this option, the user specifies a maximum base angle that can exist and SLOPE/W will force a tension crack to develop if the base angle of any of the slices exceeds this maximum value.

At the toe the soil is being pushed forward so the mechanism of a tension crack is not the correct approach to use. The critical slip surface, which results in the minimum factor of safety (F of S) will usually be close to the theoretical maximum exit angle of  $(45^\circ - \phi/2)$ .

Allowing entrance and exit angles to become much steeper than the active and passive angles can lead to convergence difficulties due to the non-linearity of the factor of safety equations. Consider the following simple example. For this case, a series of fully specified slip surfaces have been defined which vary the exit angle near the toe of the slope.



The computed factors of safety for exit inclination angles between 21.8 and 51.3 degrees are:

Exit angle	F of S
21.8	1.615
25.5	1.608
30.5 (about $45 - \emptyset/2$ )	1.604
33.6	1.610
45.9	1.633
51.3	1.673

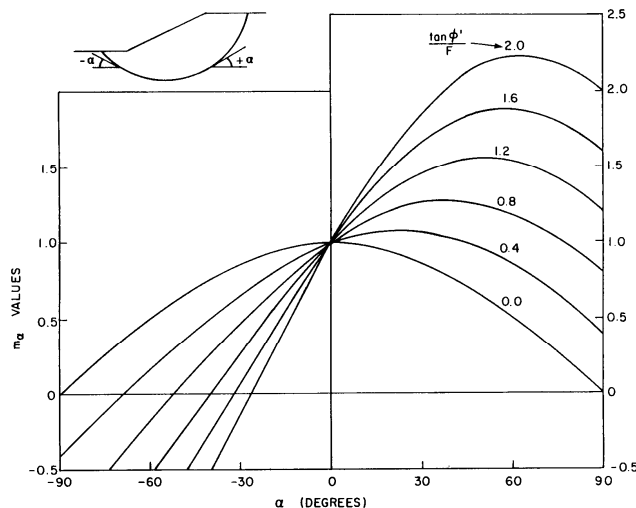
Note how the minimum value is found for that slip surface that has an exit angle close to the passive angle of  $(45^\circ - 30/2) = 30^\circ$  as it theoretically should. Also note that the F of S is really not very sensitive to the inclination angles ultimately used as there is only about a 5% change over the wide range of angles analysed.

## Undrained Behavior

It is sometimes assumed that for undrained behavior  $\emptyset$  is zero and therefore the crest (active) and toe angles (passive) should be  $45^\circ$ . This assumption is not correct. The soil does not know that  $\emptyset$  is zero. The soil is aware of only one thing and that is  $\emptyset'$  (phi effective). An undrained  $\emptyset$  of zero is only an externally observed phenomenon due to a particular loading and drainage condition. In reality,  $\emptyset'$  does not change. The active and passive slip angles are controlled by  $\emptyset'$ , not by  $\emptyset$  total. Therefore the entrance and exit angles will theoretically be  $(45^\circ + \emptyset'/2)$  and  $(45^\circ - \emptyset'/2)$  even for rapid loading and undrained conditions.

## Bishop $m_\alpha$ term

Even the Bishop simplified factor of safety equation for moment equilibrium is non-linear and Bishop recognized early on that very steep slice base inclinations at the crest and the toe could lead to numerical difficulties. Consequently, you will usually see in text books that present the Bishop formulation the following  $m_\alpha$  chart. The idea was to illustrate that convergence difficulties could creep into the analysis if the crest and toe angles were too steep.

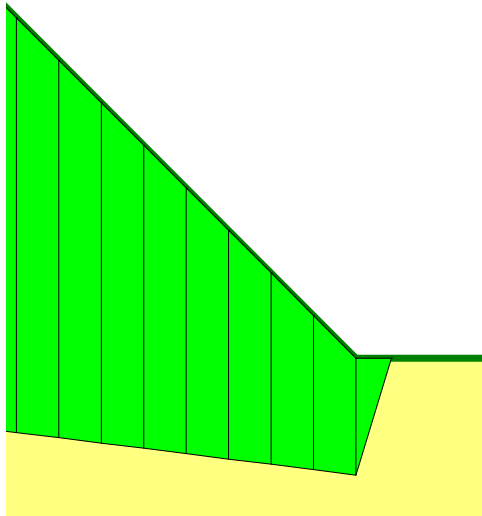


## Physical admissibility

Unfortunately it is possible to sometimes compute a solution for physically inadmissible trial slip surfaces that give unrealistic factors of safety and provide erroneous results. Safety factors for very steep exit angles for the above example are as follows:

Exit angle	F of S
51.3	1.673
59.04	1.782
68.2	No convergence
73.3	1.258
78.7	1.302
84.3	1.306

The slip surface shape that gives a F of S of 1.258 (the lowest computed) is as follows:



We can likely all agree that the soil will not jump out of the ground at such a steep angle in the toe area. In other words, this is logically not a physically admissible trial slip surface and the computed F of S, however low it may be, is consequently irrelevant. Without an exit inclination angle restriction, SLOPE/W would present this as the critical failure surface and therefore present an inappropriate result.

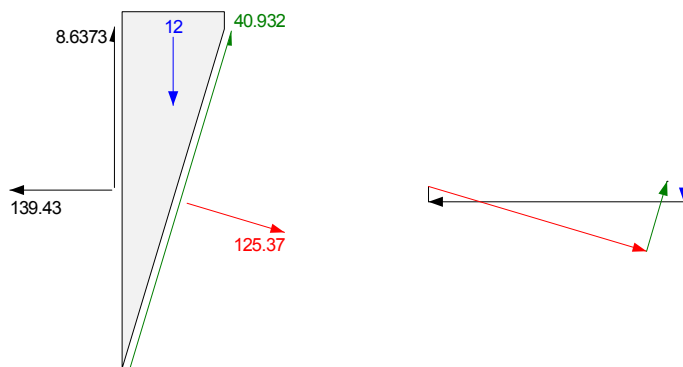
## Reason for unrealistic safety factors

The potential for unrealistic F of S values lies in the limit equilibrium formulation. Since the formulation is based only on equations of statics without any consideration of a stress/strain relationship, the LE formulation uses an iterative process until the following two conditions are achieved:

1. Each slice must be in force equilibrium.
2. The factor of safety must be the same for each slice (i.e., the F of S is constant along the slip surface).

The following diagram gives the free body diagram and force polygon for the toe slice. Note how the normal at the slice base and the normal interslice force on the left point away from the slice. The force polygon nearly closes, indicating the slice is in force equilibrium (within the convergence tolerance). The normal forces are not realistic but they are the forces that were required to meet the above two conditions.

Slice 30 - Morgenstern-Price Method



If the base normal points away from the slice, the base shear reverses and therefore reduces the overall shearing resistance which in turn leads to a lower factor of safety.

The inherent problem lies in the assumption of a constant  $F$  of  $S$  existing along the entire slip surface. In reality this is not true. Overcoming this limitation requires adding some form of a stress-strain law to the analysis (that is, by conducting a finite element analysis for example). Ways of doing this have been presented in the SLOPE/W related Engineering book called *Stability Modeling with SLOPE/W- An Engineering Methodology* in Chapter 2.

Some researchers and analysts, upon recognizing the unrealistic base normal forces, tend to arbitrarily alter the slice forces to what may seem intuitively reasonable. This violates the fundamental assumptions in the LE formation and could mean that the overall sliding mass is potentially no longer in force and moment equilibrium. Stated another way, by arbitrarily altering the slice forces, you can make the  $F$  of  $S$  suspect and perhaps even irrelevant. Even though we know the slice forces are not realistic, it is not appropriate to arbitrarily alter the slice forces. In the end the slice forces must be such that the above two criteria are satisfied.

In our view it is better to not analyze potentially physically inadmissible slip surfaces than to arbitrarily alter slice forces.

## Footing Analysis

Our experience indicates that the discussion and observations presented here apply equally to a footing (bearing pressure) analysis as to a conventional slope stability analysis.

## Concluding remarks

This technical note explains why SLOPE/W limits the inclination of the slip surface exit angle. Without the restriction in place there is a danger of presenting unrealistically low safety factors for physically inadmissible slip surfaces. Discussing this issue also highlights the need to look at more information for a slope stability analysis than just the factors of safety. Once we look at the free body diagram and force polygon for the toe slice with a very steep exit angle, we are all likely willing to dismiss the presented F of S value of 1.258 as being unrealistic. Furthermore, it is very difficult to debate issues like this if only the factor of safety is presented. More than just the factor of safety needs to be presented if one is going to argue in favor of a particular position.

The example shown above illustrates that the trial slip surface with the mathematically lowest F of S is not necessarily the governing mode of failure. The potential mode of failure and related slip surface shape must be reasonable for the F of S to be representative of actual field conditions. In the end, it is important to not forget about sound and prudent judgment when interpreting the results of a limit equilibrium stability analysis.

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(Note: the exit angle restriction was removed in the SLOPE/W computer code for preparing this discussion to illustrate the effect of very steep exit angles.)